MATH 151 Exam 4 Review Key Spring

1. Solve the following:

   (a) $y' = xe^{-sin\ x} - y\ cos\ x$

$$\text{this is a linear ODE so } \rho(x) = I(x) = e^{\int cos\ x\ dx} = e^{sin\ x}$$

$$\implies \quad [e^{sin\ x}y]' = x \quad \implies \quad y = \left(\frac{x^2}{2} + C\right) e^{-sin\ x}$$

   (b) $(3y^2 + 2y)y' = x\ cos\ x$

$$\text{this is a separable ODE so } \int (3y^2+2y)\ dy = \int x\ cos\ x\ dx \text{ note that the RHS needs parts}$$

$$\implies \quad y^3 + y^2 = cos\ x + x\ sin\ x + C$$

2. Solve the following initial value problems:

   (a) $\frac{dy}{dx} = \frac{y\ cos\ x}{1+y^2}$, $y(0) = 1$

$$\frac{1+y^2}{y}dy = cos\ x\ dx \quad \implies \quad \int \left(\frac{1}{y} + y\right)\ dy = \int cos\ x\ dx$$

$$\implies \quad ln|y| + \frac{1}{2}y^2 = sin\ x + C \text{ and since } y(0) = 1,\ C = \frac{1}{2} \quad \implies \quad ln|y| + \frac{1}{2}y^2 = sin\ x + \frac{1}{2}$$

   (b) $xyy' = ln\ x$, $y(1) = 2$

$$\text{this is a separable ODE so } ydy = \frac{ln\ x}{x}dx$$

$$\int y\ dy = \int \frac{ln\ x}{x}\ dx \quad \implies \quad \frac{y^2}{2} = \frac{u^2}{2} + C \quad \implies \quad \frac{1}{2}y^2 = \frac{1}{2}(ln\ x)^2 + C \quad y(1) = 2 \quad \implies \quad C = 2$$

$$\implies \quad y = \sqrt{(ln\ x)^2 + 4}$$

   (c) $y' + y = \sqrt{x}e^{-x}$, $y(0) = 3$

$$\text{this is a linear ODE so } \rho(x) = I(x) = e^{\int dx} = e^x$$

$$\implies \quad [e^x y]' = \sqrt{x} \quad \implies \quad y = \left(\frac{2}{3}x^{3/2} + C\right) e^{-x} \qquad y(0) = 3 \quad \implies \quad C = 3$$

$$\implies \quad y = \left(\frac{2}{3}x^{3/2} + 3\right) e^{-x}$$

(d) Solve the IVP $x\frac{dy}{dx} - y = 2x^2y \quad , \quad y(1) = 1$

$$\text{this is a separable ODE so } \frac{1}{y}dy = \left(2x + \frac{1}{x}\right) dx \quad \ldots$$

$$\Longrightarrow \quad y = \frac{xe^{x^2}}{e}$$

(e) Solve: $\quad xy' + (2x-3)y = 4x^4$

$$y' + \frac{2x-3}{x}y = 4x^3 \quad \Longrightarrow \quad \rho(x) = I(x) = e^{\displaystyle\int \frac{2x-3}{x}\, dx} = e^{2x - ln\ x^3} = \frac{e^{2x}}{x^3}$$

$$\Longrightarrow \quad y = 2x^3 + C$$

(f) Solve the IVP $y' = (1-y)cos\ x \quad , \quad y(\pi) = 2$

$$\text{this is a separable ODE so } \frac{1}{1-y}dy = cos\ xdx \quad \ldots$$

$$\Longrightarrow \quad -ln\big|1-y\big| = sin\ x \ \text{ or } \ y = 1 - e^{-sin\ x}$$