MATH 151 Exam 3 Review Key Spring

1. Find the **sum** of $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+3)}$

$$= \sum_{n=1}^{\infty} \left[\frac{1}{3n} - \frac{1}{3(n+3)}\right] \implies s_n = \left(\frac{1}{3} + \frac{1}{6} + \frac{1}{9} + \ldots - \frac{1}{3(n+1)} - \frac{1}{3(n+2)} - \frac{1}{3(n+3)}\right)$$

$$\text{so } \lim_{n\to\infty} s_n = \frac{11}{18}$$

2. Determine whether the series is conditionally convergent, absolutely convergent or divergent.

$$\sum_{n=1}^{\infty} (-1)^{n-1}\, n^{-1/3}$$

by the AST the series converges and $\sum \left|a_n\right| = \sum \frac{1}{n^{1/3}}$ is a divergent p-series $\implies CC$

3. Test the series for convergence or divergence

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\sqrt{ln\ n}}$

use the integral test $\longrightarrow D$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1\cdot 3\cdot 5\cdots(2n-1)}{5^n n!}$

use the ratio test $\longrightarrow \displaystyle\lim_{n\to\infty} \frac{1\cdot 3\cdot 5\cdots(2n-1)(2n+1)}{5^{n+1}(n+1)!}\cdot\frac{5^n\ n!}{1\cdot 3\cdots(2n-1)} = \ldots \frac{2}{5} < 1 \implies AC$

4. Test the following for absolute convergence, conditional convergence or divergence.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n-1}{n^2+n}$

LCT with $\sum \frac{1}{n} \implies D$

(b) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n-1}{n^2+n}$

by AST series converges, however $\sum \frac{n-1}{n^2+n}$ diverges by LCT $\implies CC$

(c) $\sum_{n=1}^{\infty} \frac{(3n)^n}{(1+8n)^n}$

Root test $\implies$ AC

(d) $\sum_{n=1}^{\infty} n^2 e^{-n^3}$

Integral test $\implies$ AC

(e) $\sum_{k=1}^{\infty} \frac{2^k k!}{(k+2)!}$

Ratio test or $\lim_{n\to\infty} a_n \neq 0 \implies D$

(f) $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2+25}$

by AST series converges, however $\sum \frac{n}{n^2+25}$ diverges by LCT $\implies$ CC

(g) $\sum_{n=1}^{\infty} sin\ n$

$\lim_{n\to\infty} a_n\ DNE \implies D$

(h) $\sum_{n=1}^{\infty} \frac{n^2+1}{n^3+1}$

LCT with $\sum \frac{1}{n} \implies D$

(i) $\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}-1}$

by AST series converges, however $\sum \frac{1}{\sqrt{n}-1}$ diverges by direct comparison $\implies$ CC

(j) $\sum_{k=1}^{\infty} \frac{k+5}{5^k}$

Ratio test $\implies$ AC